

## Calculated Value of Gravitational Constant from Fine Structure Constant and Electron Mass

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In the Charged Particle Interaction Section of “A Quantum Theory Conjecture on the Origin of Gravitational and Electric Particle Interaction”[1], there is a relational value between  $\alpha$  and other fundamental constants including The Gravitational Constant and the Electron mass, that can be useful in testing the theory.

$$\alpha^2 = \frac{2\hat{\lambda}_{\text{PL}}^2}{\hat{\lambda}_e^2} \nu_e^2 \quad (1)$$

This is the value of the fine structure constant, Alpha ( $\alpha$ ), in terms of fundamental constants that are known to a high degree of precision by calculations from QFT considerations. The  $\hat{\lambda}_e$  is the Compton radius of the electron, and  $\nu$  is the Compton frequency  $\nu_e = c / (2\pi\hat{\lambda}_e)$ . The Planck length is:

$$\hat{\lambda}_{\text{PL}}^2 = \frac{G\hbar}{c^3}, \quad (2)$$

The value of  $\alpha^2$  is nominally:

$$\alpha^2 = (1/137)^2, \quad (3)$$

or more accurately from the work of Gabrielse et.al.[2],

$$\alpha = 0.00729735253594 \quad (4)$$

Whether this theory has merit or not, hinges to a degree on the accuracy of the prediction of this relation. This value of  $\alpha$  (Eq.(1)), is expressed in fundamental constants known to a high accuracy (~12 significant digits) and there are no

“fudge” factors, The major uncertainty in the calculation is the value of the gravitational constant  $G$ , which is experimentally determined to only about 5 significant digits, is the major uncertainty, the gravitational constant..

A more explicit for Eq.(1), is:

$$\alpha^2 = \frac{G\hbar}{c^3} \frac{2}{\lambda_e} \frac{c^2}{2^2 \pi^2 \lambda_e^2} = \frac{G\hbar}{2\pi^2 c \lambda_e^4} \quad (5)$$

The model for the electron that generates the relation for  $\alpha$  is of two photons revolving round the center of momentum, and just as in the case of an electron revolving around the proton, the quantum loops of the Feynman paths of the orbiting photons must be taken into account. The mass calculated Compton wavelength is the first loop, and the effect on the wavelength of all the other loops must be taken into account, this is done by multiplying the Compton radius  $\lambda_e$ , by the anomalous gyromagnetic ratio  $g_e / 2$

$$g_e / 2 = 1.0011596522 \quad (6)$$

To assess the predicted theoretical value, the expression Eq.(5), can be solved for the gravitational constant, and compared with current experimental values.

Solving for the electron mass in terms of fundamental constants gives:

$$m_e = \frac{\hbar g_e}{c} \left( \frac{2\pi^2 c \alpha^2}{G\hbar} \right)^{1/4} \quad (7)$$

Solving, for the gravitational constant gives:

$$G = \frac{\alpha^2 2\pi^2 c (\lambda_e g_e)^4}{\hbar}, \quad (8)$$

Calculating G from Eq.(8), to 12 significant digits gives:

$$G = \underline{6.67586727955} \times 10^{-08} \text{ cm}^3 \text{ gm}^{-1} \text{ s}^{-2}$$

This is slightly higher (0.02%) than the current Codata consensus recommended value of  $6.67430(15) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ , , but it is not outside the scatter of current measurements, and is exactly the value determined by the Cavendish balance measurements of the International Bureau of Weights and Measures, BIPM value by T. Quinn et.al,[3], published in 2015.

$$G = \underline{6.67586(36)} \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

The calculated value is in agreement with this value to the limits of its experimental accuracy of this measurement.

### Conclusion

Accept for some numerology associations, the equation above, Eq.(1), is the only known physical relation between the fine structure constant, the Gravitational constant, and the mass of the electron. The scatter in the experimental value of G is the only uncertainty.

Reference values:

Constants used in calculation in CGS units.

$c = 2.9979245800\text{E}+10$	$g_F = g_e / 2 = 1.00115965218$
$\hbar = 1.0545918473\text{E}-27$	$\tilde{\lambda}_e = \hbar / (m_e c) = 3.8616633678\text{E}-11$
$\alpha = 1/137.03599971$	$m_e = 9.109389966\text{E}-28$

Summary:

Calculated value of G	$G = 6.67586727955 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
BIPM Cavendish balance value	$G = 6.67586(36) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Codata Consensus Value	$G = 6.67430(15) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

Excerpt from BPIM report: <https://royalsocietypublishing.org/doi/pdf/10.1098/rsta.2014.0032>

Downloaded from <http://rsta.royalsocietypublishing.org/> on November 4, 2015

## 12. A value for Newton's constant of gravitation

The peak-to-peak servo torque,  $\tau_s$ , obtained as an unweighted mean of 10 data runs was  $3.148869(94) \times 10^{-8}$  N m and using equations (9.2a) and (11.3) we can write

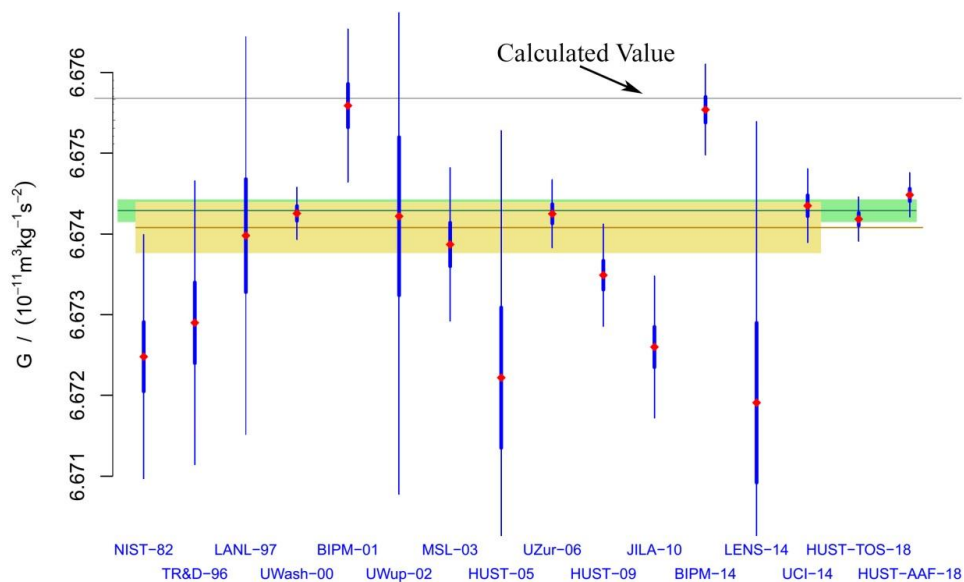
$$G_s = \frac{\tau_s}{\Gamma_s} = 6.67515(41) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \text{ (61 ppm)}. \quad (12.1a)$$

The unweighted mean of the 10 data runs giving a value of the peak-to-peak deflection angle of 0.1529322(29) mrad using equations (9.2b), (8.2) and (11.8) we can write

$$\longrightarrow G_c = \frac{\tau_c}{\Gamma_c} = 6.67586(36) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \text{ (54 ppm)}. \quad (12.1b)$$

We have used the values for the uncertainties in the experimental measurements given in §11.

### Experimental Data



Experimental values used for statistical calculation of CODATA consensus value of  $G$  [4]

## Reference:

1. D.T. Froedge, A Quantum Theory Conjecture on the Origin of Gravitational and Electric Particle Interaction, <http://www.arxdtf.org/css/QFTGravitationalElectric.pdf> DOI: 10.13140/RG.2.2.29097.54884
2. G. Gabrielse et al., New Determination of the Fine Structure Constant from the Electron  $g$  Value and QED, Phys. Rev. Lett. 97, 030802 (2006)  
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3. T. Quinn et.al, The BIPM measurements of the Newtonian constant of gravitation,  $G$   
<https://royalsocietypublishing.org/doi/pdf/10.1098/rsta.2014.0032>
4. C. Merkatas et. al., Consensus Value for the Newtonian Constant of Gravitation  
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